

**COMMON PRE - BOARD EXAMINATION - 2023**  
**MATHEMATICS (041)**

**Q P Code 65/4/1**

**MAX.MARKS: 80**

**CLASS: XII**

**TIME: 3 HOURS**

**General Instructions:**

- This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
- Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

**SECTION A**

**Choose the correct answer:**

- 1** If  $x + \sin y = \log x$ , then  $\frac{dy}{dx} =$  \_\_\_\_\_ **1**
- a)  $\frac{\log x - 1}{\cos y}$       b)  $\frac{x}{1 + \cos y}$       c)  $\frac{x-1}{x \cos y}$       d)  $\frac{1-x}{x \cos y}$
- 2** The value of  $x + y$  for which the matrix  $A = \begin{bmatrix} 0 & -3 & 1 \\ 3 & 0 & -5 \\ x & y & 0 \end{bmatrix}$  is skew symmetric, is \_\_\_\_\_. **1**
- a) 4      b) -4      c) 6      d) -6
- 3** If A is a square matrix of order 3 and  $|A| = 12$ , then the value of  $|A \text{ adj} A|$  is \_\_\_\_ **1**
- a) 12      b) 144      c) 1728      d) 27
- 4** The positions of a kite at two different timings were noted and the equation of the line joining these two points was given as  $x = -3, \frac{2y+4}{6} = 4z - 12$ . The direction ratios of the line are: **1**
- a) (1, 6, 1)      b) (1, 3, 4)      c) (0, 6, 1)      d)  $(0, 3, \frac{1}{4})$
- 5** If  $f(x) = \log \sqrt{\tan x}$ , then the value of  $f'(x)$  at  $x = \frac{\pi}{4}$  is \_\_\_\_\_ **1**
- a) 1      b) 0      c)  $\infty$       d)  $\frac{1}{2}$

- 6 If A is a square matrix such that  $A^2 = I$ , then find the value of  $(A - I)^3 + (A + I)^3 - 7A$  is **1**  
a) A b)  $I - A$  c)  $I + A$  d)  $3A$
- 7 The value of  $\lambda$  when the projection of  $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$  on  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$  is 4 units is \_\_\_\_\_. **1**  
a) 5 b) 7 c)  $\pm 5$  d)  $\pm 7$
- 8 The corner points of the feasible region determined by a system of linear inequalities with  $Z = 3x + 9y$  as objective function are A(0,20), B(15,15), C(5,5) and D(0,10). The maximum of Z: **1**  
a) occurs only at A b) occurs only at B  
c) occurs at A and B d) occurs at every point on AB
- 9 If  $|\vec{a}| = 8$ ,  $|\vec{b}| = 3$ ,  $|\vec{a} \cdot \vec{b}| = 12\sqrt{3}$ , then find the value of  $|\vec{a} \times \vec{b}|$  **1**  
a)  $4\sqrt{3}$  b)  $12\sqrt{3}$  c) 12 d) 6
- 10 If  $\begin{bmatrix} 3a+6 & 2 \\ -8 & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ , then the value of **ab** is: **1**  
a) 16 b) -16 c) 4 d) -4
- 11 The value of k for which the function  $f(x) = \begin{cases} \frac{x^3 - 8}{x - 2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$  is continuous at  $x = 2$  is \_\_\_\_\_. **1**  
a) 4 b) -4 c) 12 d) 0
- 12 The value of  $\frac{dy}{dx}$ , when  $x = a \cos^3 \theta$ ,  $y = a \sin^3 \theta$  is : **1**  
a)  $\tan \theta$  b)  $-\tan \theta$  c)  $\cot \theta$  d)  $-\cot \theta$
- 13 The corner points of the feasible region determined by a system of linear equations with  $Z = ax + by$  where  $a, b > 0$  are (0,0), (2,4), (4,0) and (0,5). The relation between a and b so that the maximum of Z occurs at both (2,4) and (4,0) is: **1**  
a)  $a = 2b$  b)  $2a = b$  c)  $a = b$  d)  $3a = b$
- 14 The derivative of  $x^x$  is **1**  
a)  $xx^{x-1}$  b)  $x^x(1 + \log x)$  c)  $x^x \log x$  d)  $x^x - \log x$

- 15 If  $P(A) = 0.4$ ,  $P(B) = 0.8$  and  $P(B/A) = 0.6$ , then  $P(A \cup B) = \text{---}$  1  
 a) 0.96                      b) 1.44                      c) 1.04                      d) 0.24

- 16 Find the degree of the differential equation:  $4 \frac{\left(\frac{d^2 y}{dx^2}\right)^3}{\frac{d^3 y}{dx^3}} + \frac{d^3 y}{dx^3} = x^2 - 1$  1

a) 2                      b) 3                      c) 1                      d) Not defined

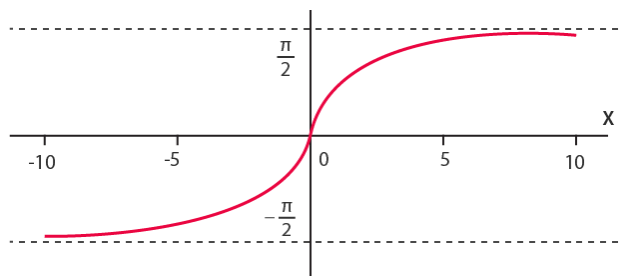
- 17 Sam plotted three points A (2, -3), B (x, -1) and C (0, 4) on a graph sheet. The value of x that makes the points collinear is \_\_\_\_\_. 1

a)  $-\frac{7}{10}$                       b)  $\frac{7}{10}$                       c)  $\frac{10}{7}$                       d)  $-\frac{10}{7}$

- 18 Identify the function from the graph. 1

a)  $\sec^{-1} x$                       b)  $\operatorname{cosec}^{-1} x$

c)  $\tan^{-1} x$                       d)  $\cot^{-1} x$



### **Assertion – Reason based questions**

In the following two questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- a) Both A and R are true but R is not the correct explanation of A.  
 b) Both A and R are true and R is the correct explanation of A.  
 c) A is true but R is false.  
 d) A is false but R is true.
- 19 **Assertion (A)** :  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x$ , then  $f$  is an injective function. 1  
**Reason (R)**: A function  $g: A \rightarrow B$  is said to be onto function if for each  $b \in B$ , there exists an  $a \in A$  such that  $g(a) = b$ .

- 20 **Assertion (A)**: If a unit vector makes angles  $60^\circ$  with  $\hat{i}$ ,  $45^\circ$  with  $\hat{j}$  and an acute angle  $\theta$  with  $\hat{k}$  then the value of  $\theta$  is  $60^\circ$ . 1

**Reason (R)**: If  $l, m, n$  are the direction cosines of a vector, then  $l^2 + m^2 + n^2 = 1$

### **SECTION B**

- 21 A man whose height is 2m walks at a uniform speed of 5m/min away from a lamp post which is 6m high. Find the rate at which the length of his shadow increases. 2

- 22 Find the length of the sum of the three mutually perpendicular unit vectors. 2

- 23 Find the domain of the function  $y = \cos^{-1}(x^2 - 4)$ . 2

**OR**

Find the value of  $\sin^{-1} \left[ \cos \left( -\frac{17\pi}{8} \right) \right]$

- 24 If  $y = \sin(\log x)$ , then show that  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$ . 2

- 25 Find the point on the line  $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$  at a distance 5 units from the point (1,3,3). 2

**OR**

If  $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$  and  $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$ , then find the value of  $\lambda$  so that  $\vec{a} + \vec{b}$ , and  $\vec{a} - \vec{b}$ , are perpendicular vectors.

### SECTION C

- 26 A shopkeeper sells three types of flower seeds,  $A_1$ ,  $A_2$  and  $A_3$ . They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability that the seed is of type  $A_2$  given that a randomly chosen seed does not germinate 3

**OR**

Two friends are playing a game by throwing a pair of dice. Find the probability distribution of number of doublets in three throws of a pair of dice

- 27 Evaluate  $\int \sqrt{\tan x} + \sqrt{\cot x} dx$  3

- 28 Find the general solution of the differential equation  $2ye^{\frac{x}{y}} dx + \left( y - 2xe^{\frac{x}{y}} \right) dy = 0$  3

**OR**

Solve the differential equation:  $\frac{dx}{dy} + \left( \frac{x}{1+y^2} \right) = \frac{\tan^{-1} y}{1+y^2}$

- 29 Evaluate  $\int \frac{\sin x}{(1 - \cos x)(2 - \cos x)} dx$  3

- 30 Solve the following LPP graphically. 3  
 Maximise:  $Z = 100x + 120y$ ,  
 Subject to constraints:  $2x + 3y \leq 30$ ;  $3x + y \leq 17$ ;  $x, y \geq 0$

- 31 Evaluate  $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$  3

**OR**

Evaluate  $\int_1^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4-x}} dx$

### SECTION D

- 32** Find the equation of the line passing through the point (1, 2, -4) and perpendicular to the lines  $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$  and  $\frac{x-15}{3} = \frac{y-29}{8} = \frac{5-z}{5}$ . Also find the equation of the line passing through the point (0, -2, 4) and parallel to the obtained line. **5**

OR

Find the coordinates of the foot of the perpendicular drawn from the point (1, 2, 1) to the line joining the points (1, 4, 6) and (5, 4, 4). Also find the image.

- 33** 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises of hard workers, the second group has honest and law abiding students and the third group contains obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of the first and second group is four times that of the third group. Using matrix method, find the number of students in each category. **5**

OR

If  $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ , and  $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ , find BA. Use this result to solve the following system of linear equations:

$$y + 2z = 7$$

$$x - y = 3$$

$$2x + 3y + 4z = 17$$

- 34** Show that the relation **R** defined in the set  $N \times N$  by  $(a, b) R (c, d) \Rightarrow a^2 + d^2 = b^2 + c^2$  where  $a, b, c, d \in N$  is an equivalence relation. **5**
- 35** Using integration, find the area of the region in the first quadrant enclosed by  $x + y = 2$ , the x axis and  $y^2 = x$ . **5**

### SECTION E

(This section comprises of 3 case study/ passage based questions with two sub parts. The first case study question has two sub parts of 2 marks each. Next two case study questions have three sub parts of marks 1, 1, 2 respectively.)

- 36** The figure alongside shows the path followed by a ball when hit by a striker in a football match. It is in the form of a parabola given by the equation  $f(x) = -16x^2 + mx + 3$ ,  $0 \leq x \leq 10$ .
- a) If 4 is the critical point of the function then find the value of m.
- b) Find the interval in which the function is strictly increasing and strictly decreasing. **4**



- 37** Indian railways is the largest rail network in Asia and the world's second largest. Hence the government pays a huge amount as fuel cost. Decreasing fuel cost can increase railway profit and hence will improve the economy.

**4**

The fuel cost for running a train is proportional to the square of the speed generated ( $v$ ) in km/hr. The fuel costs Rs 48 per hour at speed 16 km/hr and the fixed charges amount to Rs 1200 per hour.



- If  $C$  is the total cost for covering the distance  $S$  km, then express  $C$  as a function of  $v$ .
- Find the critical point for  $C$ .
- Use first derivative test to find the most economical speed of the train.

**OR**

Use second derivative test to find the point of local minimum and minimum cost if distance is 100 km.

- 38** A coach is training 3 players. He observes that player A can hit a target 4 times in 5 shots, player B can hit a target 3 times in 4 shots and player C can hit a target 2 times in 3 shots. Based on this situation answer the following questions.

**4**



- Find the probability that exactly one hits the target.
- Find the probability that exactly two of them hit the target.
- Find the probability that at most one of them hit the target.

**OR**

Find the probability that at least two of them hit the target

